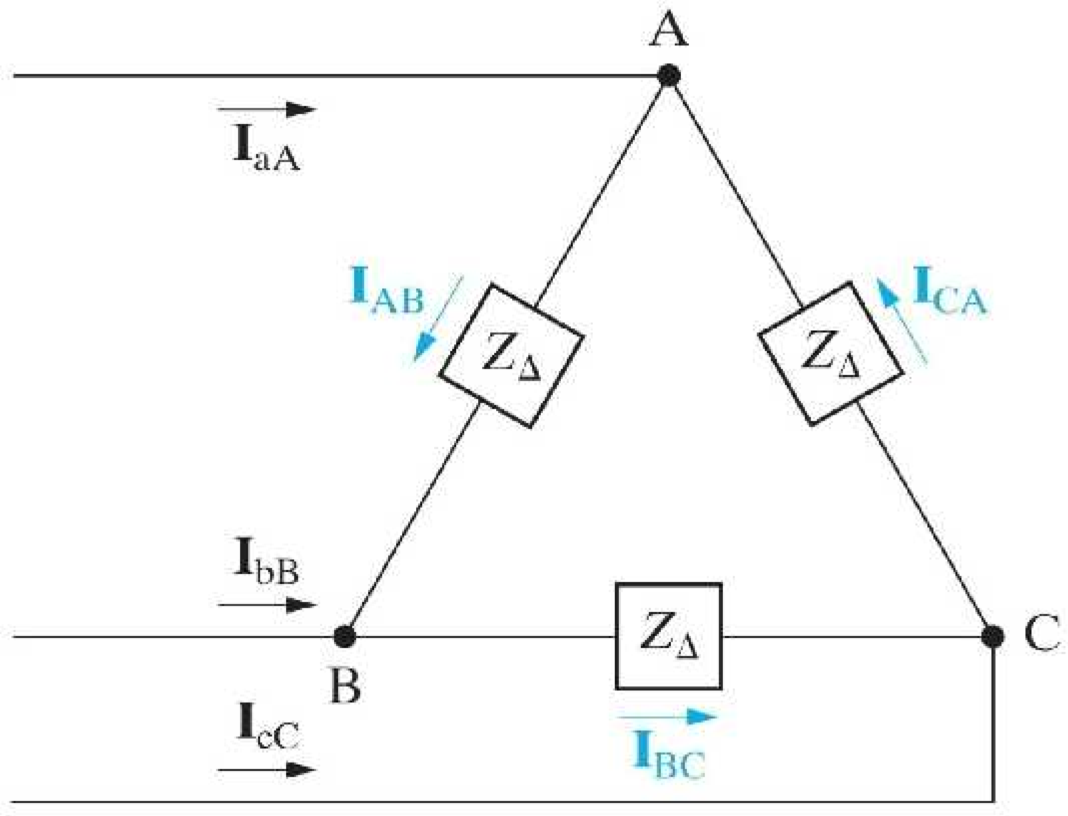
Balanced Δ-connected Load

✓ line voltages **equal** to phase voltages

$$V_L = V_\varphi$$

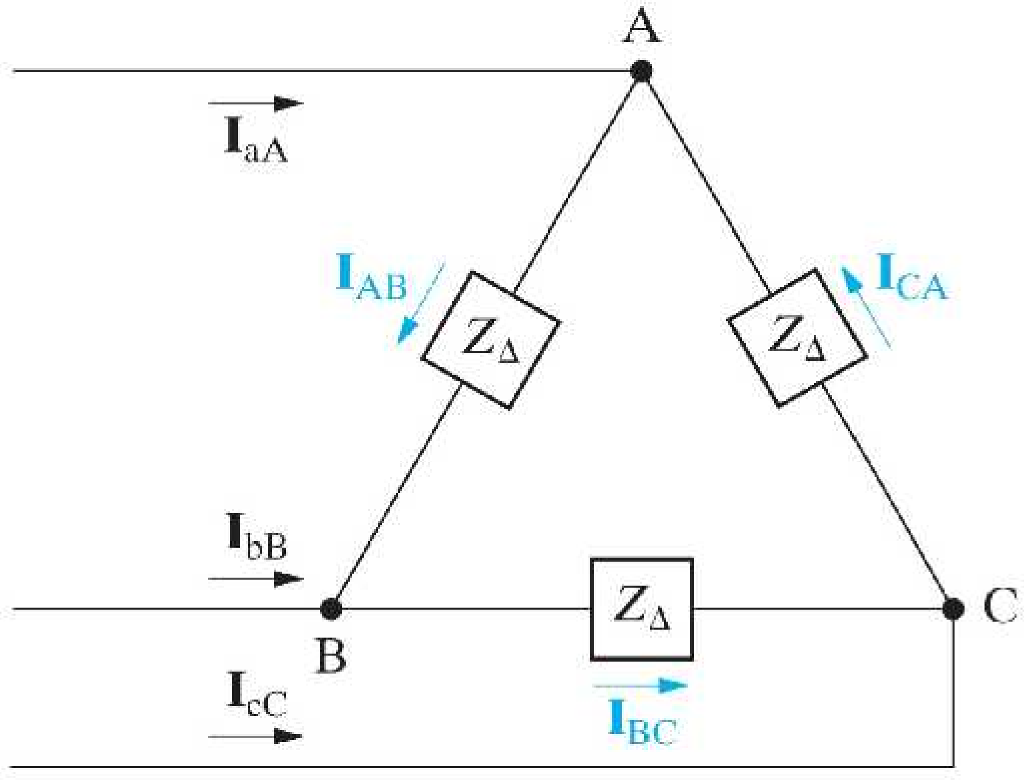


Balanced Δ-connected Load

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}}$$

$$I_{BC} = \frac{V_{BC}}{Z_{\Delta}}$$

$$I_{CA} = \frac{V_{CA}}{Z_{\Delta}}$$

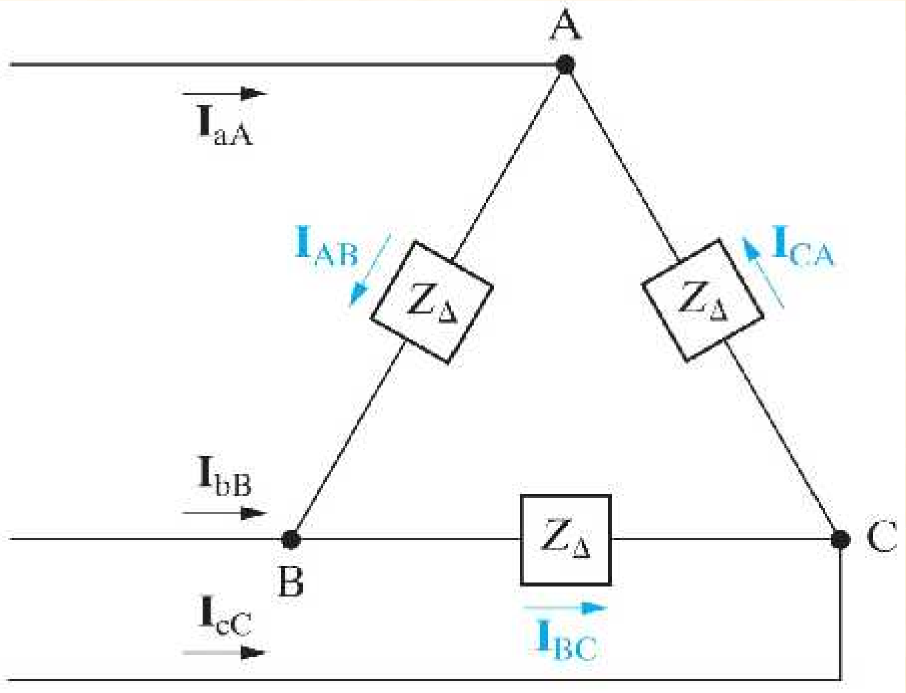


$$I_{AB} = I_{¤}\angle 0^{\circ}$$

$$I_{BC} = I_{¤}\angle -120^{\circ}$$

$$I_{CA} = I_{¤}\angle 120^{\circ}$$

Balanced Δ-connected Load



$$I_{aA} = I_{AB} - I_{CA}$$

Line Currents

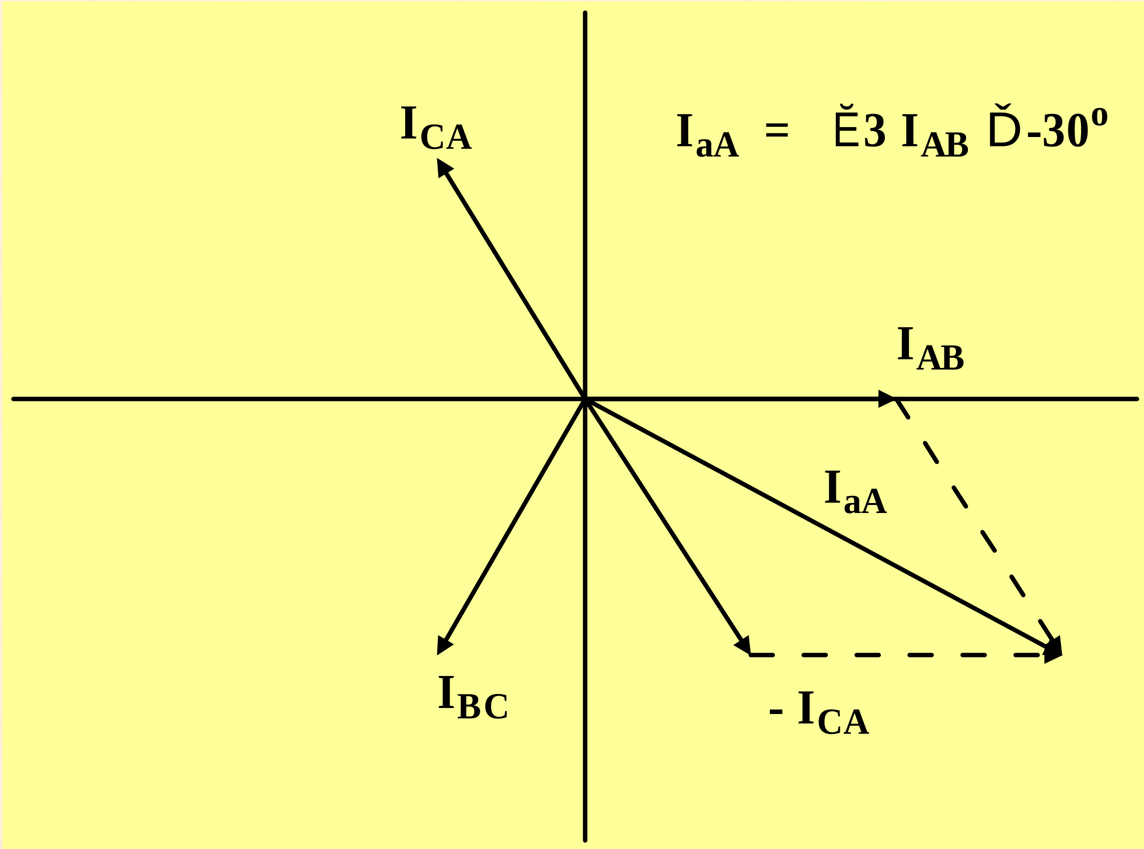
$$I_{bB} = I_{BC} - I_{AB}$$

Phase Currents

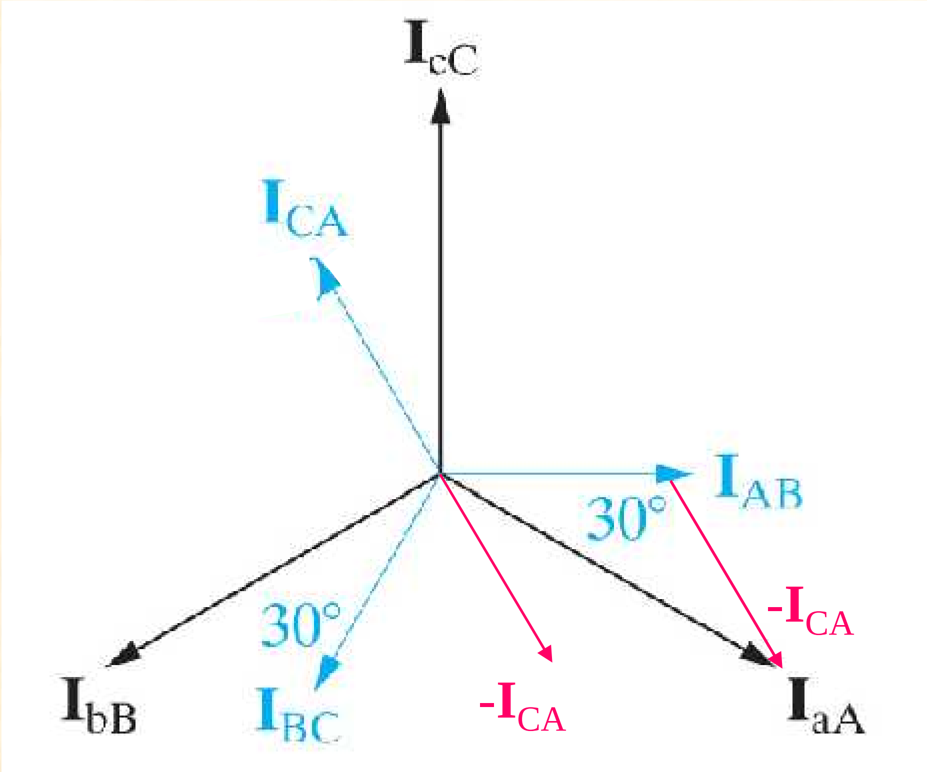
$$I_{cC} = I_{CA} - I_{BC}$$

Line and Phase Currents for Balanced Δ-connected Load

for +ve sequence



Line and Phase Currents for Balanced Δ-connected Load



for +ve sequence

I_{aA} = Line Current

I_{Φ} = Phase Current

$$I_{aA} = \left(\sqrt{3}\angle -30^\circ\right) I_{\Phi}$$

Line and Phase Currents for Balanced Δ-connected Load

PHASE CURRENTS ($I_{¤}$)

$$I_{aA} = I_{AB} - I_{CA}$$
$$I_{bB} = I_{BC} - I_{AB}$$
$$I_{cC} = I_{CA} - I_{BC}$$

$$I_{aA} = \sqrt{3}\, I_{AB} \angle -30^\circ$$
$$I_{bB} = I_{aA} \angle -120^\circ$$
$$I_{cC} = I_{aA} \angle +120^\circ$$

LINE CURRENTS (I_L)

(for +ve sequence)

Conclusions for Balanced Δ-connected Load

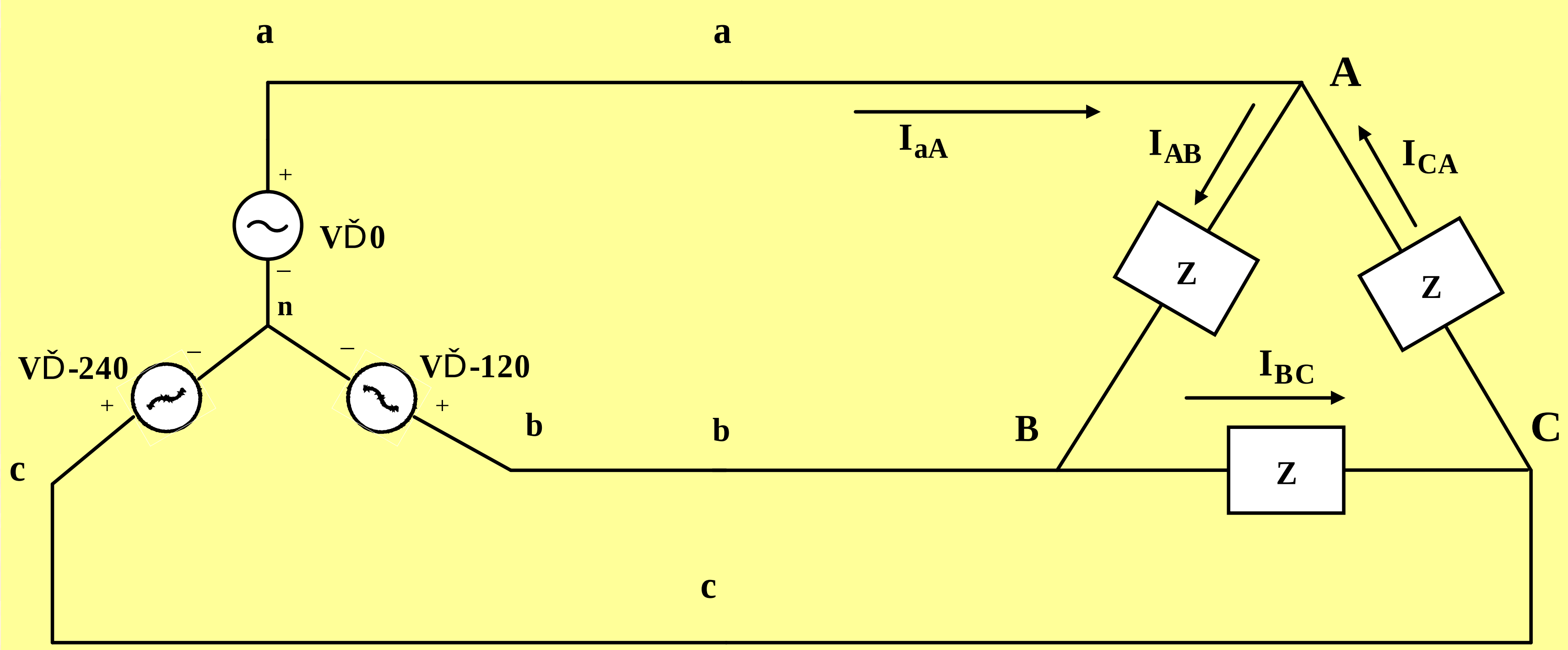
- The amplitude of the line current is equal to $\sqrt{3}$ times the phase current $\left|I_L\right| = \sqrt{3}\left|I_{¤}\right|$

- The set of line currents lags the phase currents by 30° (for +ve sequence) $\angle I_L = \angle I_{¤} - 30°$

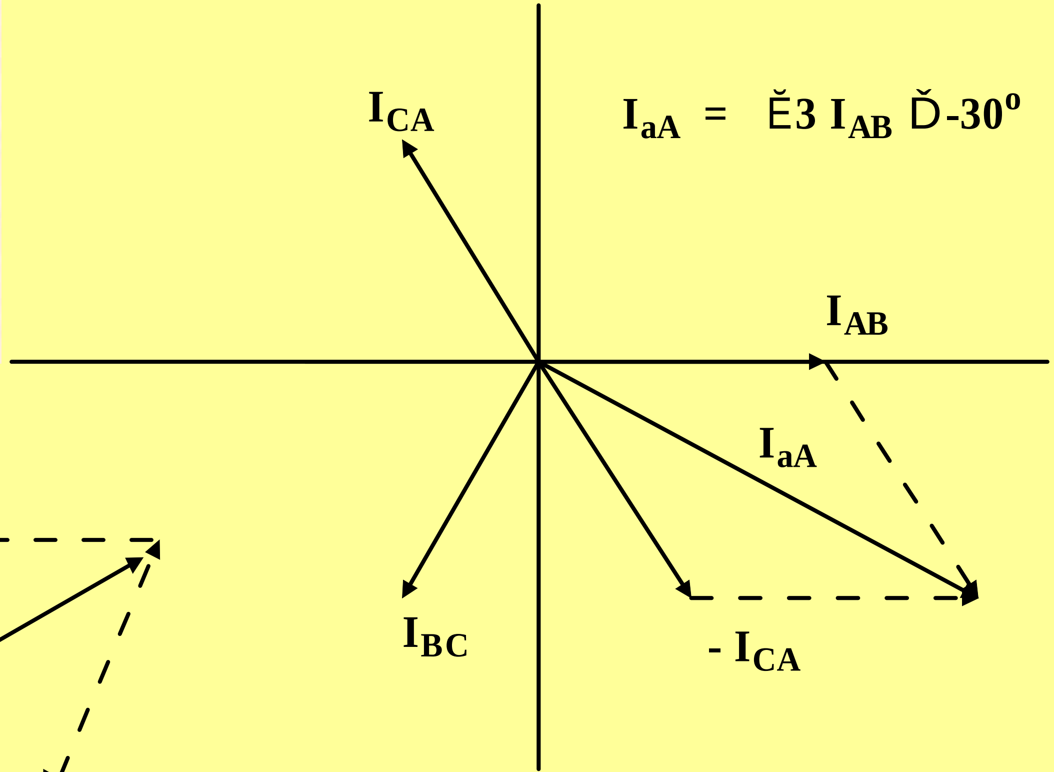
- The set of line currents leads the phase currents by 30° (for -ve sequence) $\angle I_L = \angle I_{¤} + 30°$

Balanced Y-Δ Three-Phase System

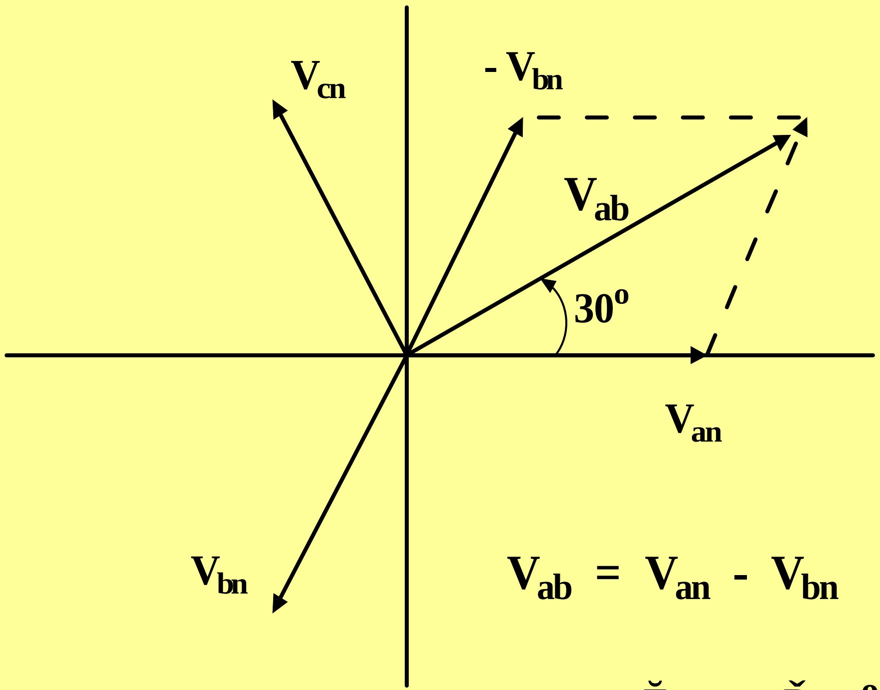
- Three phase sources are usually Y-connected and three phase loads are Delta connected.
- There is **no neutral** connection for the Y-Δ system



Balanced Y-Δ Three-Phase System



$$I_{aA} = \sqrt{3}\, I_{AB} \angle -30^o$$



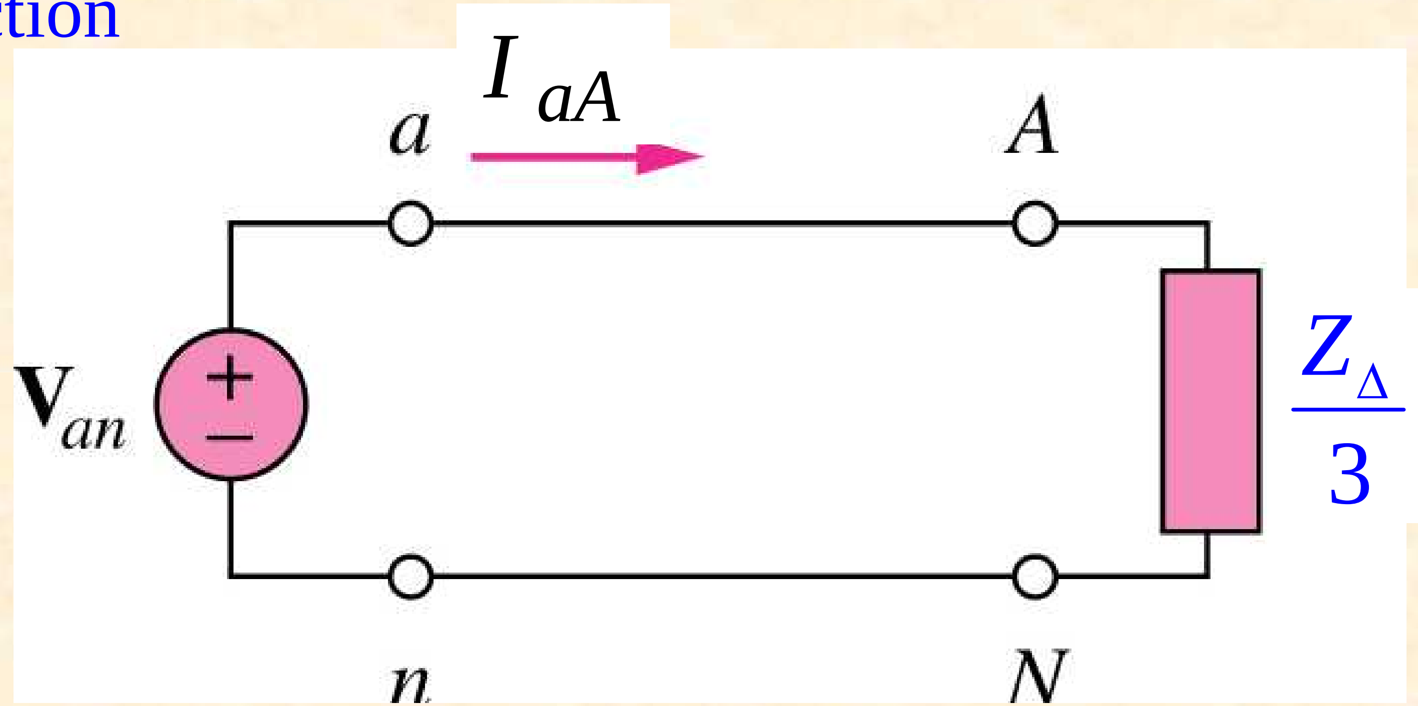
$$V_{ab} = V_{an} - V_{bn}$$

$$V_{ab} = \sqrt{3}\, V_{an} \angle 30^o$$

for +ve sequence

Balanced Y-Δ Three-Phase System

➢ Single phase equivalent circuit of the balanced **Y-Δ** connection



for +ve sequence $V_{AB} = \left(\sqrt{3}\angle 30^o\right) V_{AN}$ & $I_{AB} = I_{aA} / \left(\sqrt{3}\angle -30^o\right)$

for -ve sequence $V_{AB} = \left(\sqrt{3}\angle -30^o\right) V_{AN}$ & $I_{AB} = I_{aA} / \left(\sqrt{3}\angle 30^o\right)$

➤Example on Y- Δ System

The Y-connected source in Example 12.1 feeds a Δ-connected load through a distribution line having an impedance of $0.3 + j0.9\ \Omega/\phi$. The load impedance is $118.5 + j85.8\ \Omega/\phi$. Use the a-phase internal voltage of the generator as the reference.

a) Construct a single-phase equivalent circuit of the three-phase system.

b) Calculate the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

c) Calculate the phase voltages at the load terminals.

d) Calculate the phase currents of the load.

e) Calculate the line voltages at the source terminals.

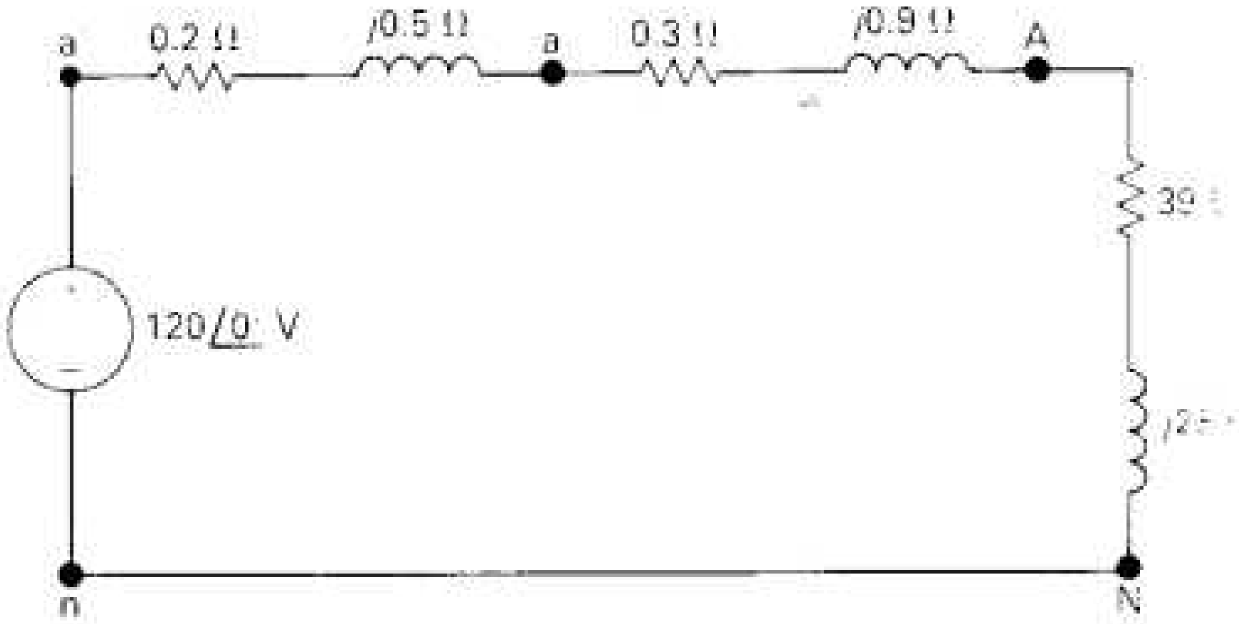


FIGURE 12.13 The single-phase equivalent circuit for Example 12.2.

a) Figure 12.13 shows the single-phase equivalent circuit. The load impedance of the Y-equivalent is

$$\left(\frac{1}{3}\right)(118.5 + j85.8) \qquad \text{or} \qquad 39.5 + j28.6\ \Omega/\phi.$$

b) The a-phase line current is

$$\mathbf{I}_{aA} = \frac{120\angle 0^\circ}{(0.2 + 0.3 + 39.5) + j(0.5 + 0.9 + 28.6)}$$

$$= \frac{120\angle 0^\circ}{40 + j30} = 2.4\angle -36.87^\circ \text{A}.$$

Hence

$$\mathbf{I}_{bB} = 2.4\angle -156.87^\circ \text{ A};$$

$$\mathbf{I}_{cC} = 2.4\angle 83.13^\circ \text{A}.$$

c) As the load is Δ-connected, the phase voltages are the same as the line voltages. To calculate the line voltages, we first calculate $\mathbf{V}_{AN}$:

$$\mathbf{V}_{AN} = (39.5 + j28.6)(2.4\angle -36.87^\circ)$$
$$= 117.04\angle -0.96^\circ \text{ V}.$$

Because the phase sequence is positive, the line voltage $\mathbf{V}_{AB}$ is

$$\mathbf{V}_{AB} = \sqrt{3}\angle 30^\circ \mathbf{V}_{AN}$$
$$= 202.72\angle 29.04^\circ \text{ V}.$$

Therefore

$$\mathbf{V}_{BC} = 202.72\angle -90.96^\circ \text{ V};$$
$$\mathbf{V}_{CA} = 202.72\angle 149.04^\circ \text{ V}.$$

d) The phase currents of the load may be calculated directly from the line currents:

$$\mathbf{I}_{AB} = \frac{1}{\sqrt{3}}\underline{/30^\circ}\,\mathbf{I}_{aA}$$

$$= 1.39\underline{/-6.87^\circ}\text{ A.}$$

Once we know $\mathbf{I}_{AB}$, we also know the other load phase currents:

$$\mathbf{I}_{BC} = 1.39\underline{/-126.87^\circ}\text{ A;}$$

$$\mathbf{I}_{CA} = 1.39\underline{/113.13^\circ}\text{ A.}$$

Note that we can check the calculation of $\mathbf{I}_{AB}$ by using the previously calculated $\mathbf{V}_{AB}$ and the impedance of the Δ-connected load. That is,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\phi} = \frac{202.72\underline{/29.04^\circ}}{118.5 + j85.8}$$

$$= 1.39\underline{/-6.87^\circ}\text{ A.}$$

(Alternative methods of calculation help eliminate errors, and we highly recommend their use in all work involving analysis and design.)

c) To calculate the line voltage at the terminals of the source, we first calculate $\mathbf{V}_{an}$. Figure 12.13 shows that $\mathbf{V}_{an}$ is the voltage drop across the line impedance plus the load impedance, so

$$\mathbf{V}_{an} = (39.8 + j29.5)2.4\angle{-36.87^\circ}$$
$$= 118.90\angle{-0.32^\circ}\text{ V}.$$

The line voltage $\mathbf{V}_{ab}$ is

$$\mathbf{V}_{ab} = \sqrt{3}\angle{30^\circ}\,\mathbf{V}_{an} \qquad \text{or} \qquad \mathbf{V}_{ab} = 205.94\angle{29.68^\circ}\text{ V}.$$

Therefore

$$\mathbf{V}_{bc} = 205.94\angle{-90.32^\circ}\text{ V};$$
$$\mathbf{V}_{ca} = 205.94\angle{+149.68^\circ}\text{ V}.$$

Common Source-Load Connection

- ✓ Common connection of source: Y
  - – **Δ**-connected sources: the circulating current may result in the delta mesh if the three phase voltages are slightly unbalanced.
- ✓ Common connection of load: **Δ**
  - – Y-connected load: neutral line may not be accessible, load can not be added or removed easily.